Collatz's conjecture
from an elementary point of view

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Abstract

One of the problems of elementary mathematics, still unsolved, is the Collatz conjecture (after the mathematician Lothar Collatz). It concerns the following function between natural numbers:
f: N →N

f(N)= { N/2 if n is even
{ 3*N+1 if n is odd

Iterating the function several times after a certain number of iterations gives the number 1 and continuing the iterations gives the cycle 1–4–2–1. Then the initial number N, in subsequent iterations, is multiplied by 3 (adding 1) and divided by 2 so that it is a succession of odd steps (when multiplying by 3 and adding 1) and even passes (when dividing by 2) which, after a certain total number of even passes b_i and a certain total number of odd steps a_i, converges to 1 based on the relation $\frac{3^{a_i}*N*Res_i}{2^{b_i}} = 1$ where is the final residue which is between Res_i**1.1** and **1.25** for reference **[1]** and therefore we will have $\frac{3^{a_i}*N}{2^{b_i}} < 1$. In this article, a new conjecture equivalent to Collatz's canonical conjecture is presented. In fact, in the following the Collatz conjecture customized by me will be applied in a faster way considering only natural numbers N modulo 4 and that the sequence of these numbers (after an initial number that can also be even mod. 4) is reduced to the sequence of only odd numbers ≡ 1 mod. 4 and ≡ 3 mod. 4. This way of proceeding leads to the same results as the canonical Collatz succession. We will indicate below with a_k the total of odd passes and with b_k the total of even passes obtained in a certain number k of successive iterations and we will indicate with N[10] the value of the number N after 10 total passes both odd and even. In addition, the term "double passage" will indicate the passage of N to $\frac{3*N+1}{2}$ that is, the odd passage always followed by an even passage. The odd numbers N ≡ 1 mod. 4 and N ≡ 3 mod. 4 will be indicated later also and respectively with N=4*h_0 +1 and N=4*h0 +3 and their subsequent ones in the Collatz sequence will be indicated with N1=4*h_1+1 and N1=4*h_1+3 ..... N_n=4*h_n+1 and Nn=4*h_n+3 where h_0, h_1, h_2,...., h_n,... we will call the coefficients of 4 of the respective odd numbers N, N1, N2,.....,Nn... ≡ 1 mod. 4 and ≡ 3 mod. 4.
Also, I will later call all even numbers N ≡ 0 mod. 4 numbers of type 1, all even numbers N ≡ 2 mod. 4 numbers of type 2, all odd numbers N ≡ 1 mod. 4 numbers of type 3 and all odd numbers N ≡ 3 mod. 4 numbers of the type 4.

1 Introduction

In my custom and faster Collatz sequence, at the beginning of each iteration you have the following 6 cases with mod numbers. 4 different from each other, of which 2 are even and 4 odd and are:

1. if the initial number N of the sequence is even with N ≡ 0 mod 4 for which it can be written $N = 2^p * (1 + 2 * k)$ for every k≥0 and p≥2 natural numbers and (1+2*k) odd integer and then for the Collatz sequence after p even single steps, in which p times N is divided by 2, leads to the number $N[p] = (2 * k + 1)$ odd and therefore to a number less than the initial number N. The number $N[p] = (2 * k + 1)$ is 2*k+1 ≡ 1 mod 4 if k is even while it is 1+2*k ≡ 3 mod 4 if k is odd and therefore in this initial iteration (from N to $N[p] = (2 * k + 1)$) odd steps are a_1**=0** while even steps are b_1**=p** (the initial difference between odd and even from zero is increased by p units, i.e. b_1-a_1**=p**);

2. if the initial number N of the sequence is even with N ≡ 2 mod 4 for which it can be written $N = 4 * k + 2$ for every k≥0 then its next in the sequence (after 1 even pass) is the odd number $N[1] = 2 * k + 1$ which is odd and less than N. If k is even 2*k+1 ≡ 1 mod 4 instead if k is odd 2*k+1

≡ 3 mod 4 and therefore in this iteration (from **N to N[1]=2*k+1**) the odd steps are $a_1=0$ while the even steps are $b_1=1$ (the difference between odd and even from zero is increased by 1 unit, i.e. $b_1-a_1=1$);

3. if the initial number N in each iteration of the sequence is odd i.e. N ≡ 1 mod 4 and can be written $N = 4 * (1 + 2 * k) + 1$ for every k≥0 (so 1+2*k is odd) then its next in the sequence (after one odd step and two even passes) is the even number $N[3] = 3 * (1 + 2 * k) + 1$ which is isopath to the odd number 1+2*k (therefore "change of orbit" and there is no longer the odd passage since $(N[3] - 1)/3 = 1 + 2 * k$) and the succession will continue with the number $N[2] = 1 + 2 * k$ odd is less than N so that in this iteration (N to $N[2] = 2 * k + 1$) Odd steps are $a_1=0$ while even steps are $b_1=2$. Furthermore, it is obvious that the number if k is even will be 1+2*k ≡ 1 mod 4 while if k is odd it will be 1+2*k ≡ 3 mod 4 (the difference between odd and even is increased by 2 units, i.e. $b_1-a_1=2$);

4. if the initial number N in each iteration of the sequence is odd i.e. N ≡ 1 mod 4 and can be written $N = 4 * (2 * k) + 1$ for every k≥0 (so 2*k is even) then its next in the sequence (after one odd pass and two even passes) is the odd number $N[3] = 3 * (2 * k) + 1 = 6 * k + 1$ which is odd and less than N so that in this iteration (from N to $N[3] = 6 * k + 1$) Odd steps are $a_1=1$ while even steps are $b_1=2$. Furthermore, it is obvious that the number 6*k+1 if k is even will be ≡ 1 mod 4 while if k is odd it will be ≡ 3 mod 4 (the difference between odd and even is increased by 1 unit, i.e. $b_1-a_1=1$);

5. if the initial number N in each iteration of the sequence is odd i.e. N ≡ 3 mod 4 and therefore N+1 is even and can be written $N + 1 = 2^p * (1 + 2 * k)$ from which $N = 2^p * (1 + 2 * k) - 1$ with **k≥0** e **p≥2**. It is shown (see **theorems 2.1 – 2.2** and **note 2.1**) that if k and p are both odd or both even, then its next in the sequence (after p-1 odd steps and p+1 even steps) is the odd number $N[2 * p] = \frac{[3^{(p-1)} * (1+2*k)-1]}{2}$ and therefore the odd passages are $a_1=p-1$ while the even passages are $b_1=p+1$ (the difference between odd and even is increased by 2 units, i.e. $b_1-a_1=2$);

6. if the initial number N of the sequence is odd i.e. N ≡ 3 mod 4 and therefore N+1 is even and can be written $N + 1 = 2^p * (1 + 2 * k)$ from which $N = 2^p * (1 + 2 * k) - 1$ for every **k≥0** and **p≥2**. It is proved (see **theorems 2.1 – 2.2** and **note 2.1**) that with k even and p odd or with k odd and p even then its next in the sequence (after p odd steps and p+1 even steps) is the odd number $N[2 * p + 1] = \frac{[3^p * (1+2*k)-1]}{2}$ and therefore the odd passages are $a_1=p$ while the even passages are $b_1=p+1$ (the difference between odd and even is increased by 1 unit, i.e. $b_1-a_1=1$).

SUMMARY TABLE OF THE 6 CASES VALID FOR EVERY k>=0 AND FOR EVERY p>=2.

Caso 1.	N ≡ 0 mod 4 --> N=2^p*(1+2*k)	N[p]=(1+2*k) --> ≡ 1 mod 4 if k is even and we continue with case 3. or case 4. N[p]=(1+2*k) --> ≡ 3 mod 4 if k is odd and we continue with case 5. or case 6.
Caso 2.	N ≡ 2 mod 4 --> N=4*k+2	N[1]=2*k+1 --> ≡ 1 mod 4 if k is even and we continue with case 3. or case 4. N[1]=2*k+1 --> ≡ 3 mod 4 if k is odd and we continue with case 5. or case 6.
Caso 3.	N ≡ 1 mod 4 --> N=4*(1+2*k)+1	N[2]=(1+2*k) --> ≡ 1 mod 4 if k is even and we continue with case 3. or case 4. N[2]=(1+2*k) --> ≡ 3 mod 4 if k is odd and we continue with case 5. or case 6.
Caso 4.	N ≡ 1 mod 4 --> N=4*(2*k)+1	N[3]=(6*k)+1 --> ≡ 1 mod 4 if k is even and we continue with case 3. or case 4. N[3]=(6*k)+1 --> ≡ 3 mod 4 if k is odd and we continue with case 5. or case 6.
Caso 5.	N ≡ 3 mod 4 --> N=2^p*(1+2*k)-1	N[2*p]=[3^(p-1)*(1+2*k)-1]/2 whether k and p are both odd or even and continue with case 3. or case 4. or case 5. or case 6.
Caso 6.	N ≡ 3 mod 4 --> N=2^p*(1+2*k)-1	N[2*p+1]=[3^(p)*(1+2*k)-1]/2 if k is even and p is odd or if k is odd and p is even and continue with case 3. or case 4. or case 5. or case 6.

It can be deduced that from the first iteration we have $b_1>a_1$ and in subsequent iterations we have that b_i increases more than a_i and that their difference in each iteration increases by 1 or 2 units compared to the previous iteration.

2 Proof of the first theorems:

Important consideration 2.1: if at the beginning of each iteration of the sequence we have the number N of the type N ≡ 3 mod 4, which can be written $N = 2^p * (1 + 2 * k) - 1$ with **k≥0** and **p≥2**, it has p consecutive double steps, i.e. all the steps of the type (3*N+1)/2 that have an odd step (multiply by 3 and add 1) always followed by a "single" even step (divide by 2) and so on. For example $N = 2127 = 4 * 531 + 3$ type N ≡ 3 mod 4 its next is $N + 1 = 2128 = 2^4 * 133$ then (1+2*k)=133 and p=4 and note that N=2127 in its Collatz sequence has 4 double passages, i.e. N[2]=(3*2127+1)/2=3191 → N[4]=(3*3191+1)/2=4787 → N[6]=(3*4787+1)/2=7181→ N[8]=(3*7181+1)/2=10772 and then you have an even single pass → N[9]=10772/2=5384. It can be seen that after the 9 passes, of which 4 doubles and one even pass, the number obtained N[9]=5384 is greater than the initial number N[0]=2127.

Important consideration 2.2: the third type N ≡ 1 mod 4 (i.e. N=4*h_0+1) leads after a double pass and a single pass to the number N[3]=3*h0+1 which in the case where h0 is odd the number N[3]=3*h_0+1 is isopath to the number N'= h0 since h_0 is odd the next of N'= h_0 as a single pass is N'[1]=3*h_0 + 1. So the initial number N=4*h_0+1 and N'= h_0 if h_0 is odd converge, in the Collatz sequence, both at the number N[3]=3*h_0+1 (so N and N' are called isopath). In this case we will say that the initial number N=4*h_0+1 and N'= h_0 continue with the orbit of N[3]=3*h_0+1 and therefore if one of them satisfies the conjecture the other 2 also satisfy it. If this happens, we will say that there has been a "change of orbit". If, on the other hand, h_0 is even, there is no change of orbit and the sequence continues with N[3]=3*h_0+1.

From the above considerations we can say that only the next of the number N ≡ 3 mod 4 (i.e. N=4* h_0 +3) in the Collatz sequence increases while all the other mod. 4 after a maximum of 2 double passes are less than the initial number N.

Theorem n.2.1 Every number $N = 2^p * (1 + 2 * k) - 1$ with k≥0 and p≥2 of the type N ≡ 3 mod 4 is transformed into the Collatz sequence in the number $N[2 * p - 2] = 3^{(p-1)} * 2 * (1 + 2 * k) - 1$ after p-1 double passes.

Demonstration. The number $N = 2^p * (1 + 2 * k) - 1$ is odd and we apply to N the double step for which $N[2] = \frac{3*(2^p*(1+2*k)-1)+1}{2} = 3 * 2^{(p-1)} * (1 + 2 * k) + \frac{-3+1}{2} = 3 * 2^{(p-1)} * (1 + 2 * k) - 1$ which is odd so we apply the double step again (in fact at each double pass the exponent of 3 increases by 1, the exponent of 2 decreases by 1 and (1+2*k) and -1 remain) and therefore we have $N[4] = 3^2 * 2^{(p-2)} * (1 + 2 * k) - 1$ which is odd and then the double step will apply until the exponent of 2 (i.e. p-2) becomes 1 and then the exponent of 3 becomes p-1 and consequently $N[2 * (p - 1)] = 3^{(p-1)} * 2 * (1 + 2 * k) - 1$. **c.v.d.**

Let's verify the previous theorem with examples. Let's consider the number N=4*37+3=151 for which $N + 1 = 152 = 2^3 * 19$ so p=3 and (1+2*k)=19. For the previous theorem, after p-1=2 double steps, we will have the number $N[4] = 3^2 * 2 * 19 - 1 = 341$ and in fact we apply the 2 double passes to N=151 so N[2]=(3*151 +1)/2=227 → N[4]=(3*227 +1)/2=341 you have the same result.
Let's consider the number again N=4*1839+3=7359 whereby $N + 1 = 7360 = 2^6 * 115$ so p=6 and (1+2*k)=115. For the previous theorem, after p-1=5 double steps, we will have the number $N[10] = 3^5 * 2 * 115 - 1 = 55889$ and in fact we apply the 5 double steps to N=7359 then N[2]=(3*7359+1)/2=11039 → N[4]=(3*11039+1)/2=16559 → N[6]=(3*16559+1)/2=24839 → N[8]=(3*24839 +1)/2=37259 → N[10]=(3*37259 +1)/2=55889 you have the same result.

Theorem n.2.2 Every number $N = 2^p * (1 + 2 * k) - 1$ with k≥0 and p≥2 of the type N ≡ 3 mod 4 transformed into $N[2 * p - 2] = 3^{(p-1)} * 2 * (1 + 2 * k) - 1$ from theorem n.2.1 is of the type ≡ 1 mod 4 that we can write N[2*(p-1)] =4*h_1+1 in conjecture. It proves N[2*p-1]=3* h_1+1 if h_1 is even or turns into N[2*p]= h_1 if h_1 is odd with $h_1 = \frac{3^{(p-1)}*(1+2*k)-1}{2}$.

Demonstration. The number $N[2 * p - 2] = 3^{(p-1)} * 2 * (1 + 2 * k) - 1$ which results after the application of theorem no. 2.1 is odd so it turns out to be of type 3 or 4. It cannot be of type 4, i.e. 4* h_1+3 since from $N[2 * p - 2] = 3^{(p-1)} * 2 * (1 + 2 * k) - 1$ = 4* h_1+3 one would have $h_1 = \frac{3^{(p-1)}*2*(1+2*k)-4}{4} = 3^{(p-1)} * \frac{1+2*k}{2} - 1$

Which is impossible because $\frac{3^{(p-1)}*(1+2*k)}{2}$ is decimal in that it is $3^{(p-1)}$ that **(1+2*k)** are odd and therefore their product is odd and not divisible by 2. So it is of type 3 because from $N[2 * p - 2] = 3^{(p-1)} * 2 * (1 + 2 * k) - 1 = 4 * h_1 + 1$ we have $h1 = \frac{(3^{(p-1)}*2*(1+2*k)-2)}{4} = \frac{(3^{(p-1)}*(1+2*k)-1)}{2}$ which is whole insofar as $3^{(p-1)} * (1 + 2 * k)$ is

odd and subtracting 1 becomes even so it is divisible by 2. Therefore, for the previous Important Consideration 2.2 we have that the transformed by theorem n.1 becomes $N[2*(p-1)+3] = N[2*p+1] = 3*h_1+1$ herself h_1 is even or turns into $N[2*(p-1)+2] = N[2*p] = h_1$ herself h_1 is odd with $h_1=\frac{(3^{(p-1)}*(1+2*k)-1)}{2}$ **.c.v.d.**

Let's verify theorem no. 2.2 above with examples. Let's consider the number N[0]=4*37+3=151 for which $N[0]+1 = 152 = 2^3*19$ so p=3 and (1+2*k)=19 which by theorem n.2.1 after p-1=2 double steps in the Collatz sequence becomes the number $N[4] = 3^2*2*19-1 = 341$ which is of the type 4* h1 +1 with h_1 =(341-1)/4=85 which is odd so that there is a change of orbit and N[6]=85 and therefore N[7]=3*85+1=256. If we continue the sequence normally with N[4]=341 we have N[6]=(3*341+1)/2=1024 from which N[8]=256 and therefore we verify that the numbers 341 and 85 are isopaths to each other because both have the number 256 in their sequences. Let's consider again the number N[0]=4*1839+3=7359 for which $N[0]+1 = 7360 = 2^6*115$ so p=6 and (1+2*k)=115 which for theorem n.1 after p-1=5 double steps in the Collatz sequence becomes the number $N[10] = 3^5*2*115-1 = 55889$ which is of the type 4* h1 +1 with h_1 =(55889-1)/4=13972 which is even for which we have N[13]=3*13972+1=41917. If we continue the sequence normally with N[10]=55889 we have N[12]=(3*55889+1)/2=83834 which is even from which N[13]=83834/2=41917.

Note 2.1: So from **theorems 2.1 and 2.2** we have that every number N[0]=4*h_0+3 type 4 with $N[0]+1 = 2^p*(1+2*k)$ where p is the periodicity of N[0]+1 with respect to 2 and (1+2*k) odd positive integer turns into N[2*p+1]=3*h_1+1 herself h_1 is even or turns into N[2*p]= h1 if h1 is odd where $h_1=\frac{(3^{(p-1)}*(1+2*k)-1)}{2}$. Therefore, at the beginning of the Collatz sequence or during the same, every time we obtain an odd number of type 4 with multiplicity p followed, as we know, by an odd number of type 3 using the previous theorems, we have an increment of odd steps equal to **p** and an increment of even steps equal to **p+1** if the number of type 3 is transformed into 3* h_1 +1 with h_1 or an odd step increment equal to **p-1** and an even step increment equal to **p+1** if the type 3 number turns into h_1 odd. We also know that h_1 becomes $h1 = \frac{(3^{(p-1)}*(1+2*k)-1)}{2} = k*3^{(p-1)} + \frac{(3^{(p-1)}-1)}{2}$ i.e. the sum of two addends, of which the first $k*3^{(p-1)}$ can be odd or even if k is odd or even while the second addend $\frac{(3^{(p-1)}-1)}{2}$ it is odd or even if p-1 is odd or even respectively (in fact, if p-1=2 even we have 3^2-1)/2=(9-1)/2=4 even if instead p-1=3 odd we have (3^3-1)/2=(27-1)/2=13 odd) therefore:

a) if **k and p-1 are both even or both odd (or if k and p are one even and the other odd)** we have that **h_1 è pari** so the next type of number for Theorem 2.2 is $N[2*(p-1)+3] = 3*h_1+1$ namely $N[2*p+1] = 3*h_1+1$;
b) **if k and p-1 are one even and the other odd or vice versa (or if k and p are both even or both odd)** we have that **h_1 is odd** so the next type of number for Theorem 2.2 is $N[2*(p-1)+3] = h_1$ namely $N[2*p+1] = h_1$.

Theorem n.2.3 Every number of the type N=4*h0+3 is transformed by theorems n.2.1 and n.2.2 into an odd number of the type 3 and precisely in N[2*(p-1)+3]=3*h_1+1 herself h_1 is even or turns into N[2*(p-1)+2]= h_1 herself h_1 is odd. It is shown that $h_1 = \frac{[3^{(p-1)}*(1+2*k)-1]}{2}$ and that if k and p-1 are both even or both odd we have $N[2*p+1] = \frac{[3^{(p)}*(1+2*k)-1]}{2}$ while if k is even and p-1 is odd or vice versa we have $N[2*p] = \frac{[3^{(p-1)}*(1+2*k)-1]}{2}$.

Demonstration. From theorem no. 2.1 we know that $N[2*p-2] = 3^{(p-1)}*2*(1+2*k)-1$ and from theorem n.2.2 we have that is of the type 3 i.e. it will be equal to 4*h_1+1 and we have $N[2*p-2] = 3^{(p-1)}*2*(1+2*k)-1$ = **4*h_1+1** from which we derive $h_1 = \frac{[3^{(p-1)}*2*(1+2*k)-2]}{4} = \frac{[3^{(p-1)}*(1+2*k)-1]}{2}$. Furthermore, from **note 2.1** by substituting the value of h_1 we have that if k and p-1 are both even or both odd then it is $N[2*p+1] = 3*h_1+1 = 3*\frac{[3^{(p-1)}*(1+2*k)-1]}{2}+1 = \frac{[3^{(p)}*(1+2*k)-3]}{2}+1 = \frac{[3^{(p)}*(1+2*k)-1]}{2}$ while if k and p-1 are one even and the other odd or vice versa we have $N[2*p] = h_1 = \frac{[3^{(p-1)}*(1+2*k)-1]}{2}$. **c.v.d.**

Theorem n.2.4 In any sequence if we obtain a number of the type N=3*h_1+1 with h_1 equal and multiple of 4 it is of the type 3 i.e. N=4*h_2+1 while if h_1 even and multiple of 2 it is of the type 4 i.e. N=4*h_2+3. So in the first case we have h_2=3* h_1/4 while in the second case we have that h_2=3* h_1/4-1/2.

Demonstration. In fact, from the equality **3*h1+1=4*h2 +1** we get the following equality **h2=3*h1/4 and** from the equality **3*h1+1=4*h2 +3** we get the following equality **h2=3*h1/4-1/2..c.v.d.**

Theorem n.2.5 In any sequence if we obtain a number of the type N= h_1 with h_1 odd it is of type 3, i.e. N=4*h_2+1 herself h_1 it is subsequent to a multiple of 4 or of the type 4, i.e. N=4*h_2+3 herself h_1 is earlier than a multiple of 4. So in the first case we have in the h_2=(h_1-1)/4 while in the second case we have that h_2=(h_1-3)/4 .

Demonstration. In fact, from the equality h_1 =4* h_2+1 we obtain the following equality h2=(h1-1)/4 and from the equality h1 =4* h2+3 we obtain the following equality h2=(h1-3)/4.**c.v.d.**

3 Custom Collatz Succession

In the custom Collatz sequence after any even leading numbers **N=4*h_0 +0 or N=4*h_0 +2 we will have odd numbers of the type N=4*h_1 +3 or of the type N=4*h_1 +1 which, according to the previous theorems, are always transformed into odd numbers of the type 3*h_2 +1 with h_2 even, or odd of the type h_2 which in turn are odd numbers of the type N=4*h_3 +3 or of the type N=4*h_3 +1 and so on in each iteration. In fact,** in the custom Collatz sequence, the following cases occur individually in each iteration **:**

a) if you start the sequence with an even number N ≡ 0 mod.4 of the type $\boldsymbol{N = 2^p * (1 + 2 * k)}$ for case 1. of the introduction in the next iteration we have N[p]=(1+2*k) odd and less than the initial number N. The number N[p]=(1+2*k) it is 1+2*k ≡ 1 mod 4 if k is even while it is 1+2*k ≡ 3 mod 4 if k is odd and then in this initial iteration (N to N[p]= 2*k+1) the odd passages are ***a=0*** while the even passages are ***b=p*** (the initial difference between odd and even from zero is increased by p units, i.e. ***b-a=p***) and we then continue with cases 3. or 4. or 5. or 6. of the introduction considering all the previous theorems;

b)if, on the other hand, the sequence is started with an even number N ≡ 2 mod.4 of the type $\boldsymbol{N = 4 * k + 2}$ for case 2. of the introduction in the next iteration $\boldsymbol{N[1] = 2 * k + 1}$ which is odd and less than N. If k is even 2*k+1 ≡ 1 mod 4 instead if k is odd 2*k+1 ≡ 3 mod 4 and therefore in this iteration (from **N to N[1]=2*k+1**) the odd steps are ***a=0*** while the even steps are ***b=1*** (the difference between odd and even from zero is increased by 1 unit, i.e. ***b-a=1***) and then continue with cases 3. or 4. or 5. or 6. of the introduction considering all the previous theorems;

b) if, on the other hand, the sequence begins with an odd number N ≡ 1 mod.4 with $\boldsymbol{N = 4 * (1 + 2 * k) + 1}$ for case 3. of the introduction in the next iteration we have the odd and less number of N i.e. N[2]=1+2*k so in this iteration (from N to N[2]=2*k+1) the odd steps are ***a=0*** while the even steps are ***b=2***. Furthermore, if k is even it will be 1+2*k ≡ 1 mod 4 and then continue with 3 cases. or 4. of the introduction while if k is odd it will be 1+2*k ≡ 3 mod 4 and then continue with cases 5. or 6. of the introduction considering all the previous theorems (the difference between odd and even is increased by 2 units, i.e. ***b-a=2***);

c) if, on the other hand, the sequence begins with an odd number N ≡ 1 mod.4 with $\boldsymbol{N = 4 * (2 * k) + 1}$ for case 4. of the introduction in the next iteration we have the odd number and less than N, i.e. N[3]=3*(2*k)+1=6*k+1 so that in this iteration (from N to N[3]=6*k+1) the odd steps are ***a=1*** while the even steps are ***b=2***. Furthermore, if k is even it will be N[3]=6*k+1 ≡ 1 mod 4 and then continue with cases 3. or 4. of the introduction while if k is odd it will be N[3]=6*k+1 ≡ 3 mod 4 and then continue with cases 5. or 6. of the introduction considering all the previous theorems (the difference between odd and even is increased by 1 unit, i.e. ***b-a=1***);

d) if, on the other hand, the sequence is started with an odd number N ≡3 mod.4 with $\boldsymbol{N = 2^p * (1 + 2 * k) - 1}$ with **k≥0** and **p≥2** we have for Theorem n.2.3 in the next iteration the number $\boldsymbol{N[2 * p] = \frac{[3^{(p-1)} * (1+2*k)-1]}{2}}$ odd is greater than N if **k is even and p-1 is odd or vice versa** (the difference between odd and even is increased by 2 units, i.e. ***b-a=2***) and then continues with cases 3. or 4. or 5. or 6. of the introduction considering all the previous theorems.

e) if,on the other hand,the sequence is started with an odd number N ≡3 mod.4 with $\boldsymbol{N = 2^p * (1 + 2 * k) - 1}$ with **k≥0** and **p≥2** we have for Theorem n.2.3 in the next iteration the number $\boldsymbol{N[2 * p + 1] = \frac{[3^{(p)} * (1+2*k)-1]}{2}}$ odd and greater than N if **k and p-1 are both even or both odd** (the difference between odd and even is increased by 1 unit, i.e. ***b-a=1***) and then continues with cases 3. or 4. or 5. or 6. of the introduction considering all the previous theorems.

Note 3.1. Now let's consider any iteration **i** of the sequence with an odd initial number N ≡ 1 mod.4 or N ≡ 3 mod.4 and denote with $\mathbf{T_{1i}}$ the sum of all the previous iterations including the iteration considered in which

there was an increase of 1 unit in the difference between ***b-a*** (points d) and f) preceding iterations) and we indicate by $\mathbf{T_{2i}}$ the sum of all the iterations including the iteration considered in which there was an increase of 2 units in the difference between ***b-a*** (points c) and e) preceding) then in the iteration considered we have the following equality $b_i - a_i = T_{1i} + 2*T_{2i}$ where b_i is the total of even steps and a_i is the total of odd steps in all iterations including the considered iteration. In fact, if I indicate with $\sum_1^i(p-1)$ The sum of all the values of **p-1** (**p** is the multiplicity of 2 in the numbers transformed in the sequence deriving from numbers ≡ 3 mod.4 up to and including iteration **I**) we have that $b_i = \sum_1^i(p-1) + 2* T_{1i} + 2*T_{2i}$ ed $a_i = \sum_1^i(p-1) + T_{1i}$ whereby $b_i - a_i = \sum_1^i(p-1) + 2* T_{1i} + 2*T_{2i} - \sum_1^i(p-1) - T_{1i} = T_{1i} + 2*T_{2i}$.

If, on the other hand, the initial number N ≡ 0 mod.4 is even with $N = 2^p * (1 + 2 * k)$ then $b_1 - a_1 = p$ and we continue in the sequence with the odd transforms of N so that to the initial difference **p** in each subsequent iteration **i** is added 1 or 2 according to the corresponding transformation for which $b_i - a_i = p + T_{1i} + 2*T_{2i}$.

If, on the other hand, the initial number N ≡ 2 mod.4 is equal with $N = 4 * k + 2$ then $b_1 - a_1 = 1$ and we continue in the sequence with the odd transforms of N so that to the initial difference **1** in each subsequent iteration **i is** added 1 or 2 according to the corresponding transformation for which $b_i - a_i = 1 + T_{1i} + 2*T_{2i}$.

We verify equality $b_i - a_i = T_{1i} + 2*T_{2i} = 21$ with **i=13** based on the custom sequence in the first 13 iterations considering the last 2 columns of **Table 1** below where the initial number N=7359 is of the type N ≡ 3 mod.4.

We also check the values of a_i and b_i in the iteration **i=13** (last row) where $\sum_1^i(p-1)$**=25** so what $b_1 = \sum_1^i(p-1) + 2* T_{1i} + 2*T_{2i} = 25+2*5+2*8 = 51$ and $a_i = \sum_1^i(p-1) + T_{1i} = 25+5 = 30$.

Table 1

Iterations	step	N	Periodicity p-1 or odd and even numbers	Total odd	Total even	Type of N and its subsequent transformation	4*h0+1-->3*h0+1 or 4*h0+3-->3*h1+1 $=T_{1i}$	4*h0+1-->h1 or 4*h0+3-->h1 $=T_{2i}$	$T_{1i}+2*T_{2i}$	b_i-a_i
		7359								
1	13	41917	5	6	7	4*h0+3-->3*h1+1	1	0	1	1
2	15	10479		6	9	4*h0+1-->h0	1	1	3	3
3	24	26527	3	10	14	4*h0+3-->3*h1+1	2	1	4	4
4	35	100723	4	15	20	4*h0+3-->3*h1+1	3	1	5	5
5	39	37771	1	16	23	4*h0+3-->h1	3	2	7	7
6	44	42493	1	18	26	4*h0+3-->3*h1+1	4	2	8	8
7	46	10623		18	28	4*h0+1-->h0	4	3	10	10
8	60	30253	6	24	36	4*h0+3-->h1	4	4	12	12
9	62	7563		24	38	4*h0+1-->h0	4	5	14	14
10	67	8509	1	26	41	4*h0+3-->3*h1+1	5	5	15	15
11	69	2127		26	43	4*h0+1-->h0	5	6	17	17
12	77	1795	3	29	48	4*h0+3-->h1	5	7	19	19
13	81	673	1	30	51	4*h0+3-->h1	5	8	21	21
		Totale=	25							

To sum up, we have that in this personalized sequence the following statements are true:

i. the transformation of numbers in each iteration faithfully follows only one of the 6 cases described above (a-b-c-d-e-f);
ii. from the first iteration the total of even steps (which I indicate with b_i) is always greater than the total of odd passages (which I indicate with a_i) that is, we have $b_i > a_i$ and in subsequent iterations we have that b_i increases more than a_i;
iii. the difference between the total of even and odd passes always increases by 1 or 2 units in each iteration and this produces in subsequent iterations a difference between b_i and a_i that always increases;
iv. In each iteration **i** we have that the difference between b_i and a_i (where b_i is the total of equal and a_i is the total of odd steps in all previous iterations including the considered iteration **i**) is always the same $T_{1i} + 2*T_{2i}$ namely $b_i - a_i = T_{1i} + 2*T_{2i}$ if N is odd or always equal $b_i - a_i = p + T_{1i} + 2*T_{2i}$ if N is equal to $N = 2^p * (1 + 2 * k)$ or it is always the same $b_i - a_i = 1 + T_{1i} + 2*T_{2i}$ if N is equal to $N = 4 * k + 2$ and that as the number of iterations increases, the difference in $b_i - a_i$.

4 Example of a custom Collatz sequence.

In the following **Table 2** as an example there is the sequence of the initial number N=4*1839+3=7359 with the succession of odd numbers of type 3 and 4 where in column 1 there is the number of iterations. In column 2 there is the sum of the total odd steps with the total even passes up to the iteration just performed. In column 3 there is the sequence of the number N. In column 4 the single number indicates p-1 i.e. the multiplicity of 2 of N+1 decreased by 1 when the number is of type 4 for which there are p-1 even numbers and p-1 odd numbers or the symbol "0d2p" indicates that type 3 is transformed into odd h_0 for which there

are 2 even numbers and 0 odd numbers or the symbol "1d2p" indicates that type 3 is transformed into 3* h_0+1 with h_0 even so there are 2 even numbers and 1 odd numbers. In column 5 there are values of (1+2*k) in the case that the number is of type 4. In column 6 there are the values k (for which k is odd or even) in the case that the number is of type 4. In column 7 there is the total number of odd passages and in column 8 there is the total number of even passages since the beginning of the sequence. In column 9 there are the values of initial h_0 of the number of type 3 or of type 4 or of the odd h_0 only. In column 10 there are the values of h_1 transformed in the iteration of the number of type 3 or of the number type 4 or of the odd h_1 only. In column 11 there is the transformation undergone in the iteration of the input number: "4* h_0+1-->3* h_0+1" indicates that type 3 turns into 3* h_0+1 while "4* h_0+1--> h_0" indicates that type 3 turns into h_0 because it is odd while "4* h_0+3-->3* h_1+1" indicates that type 4 turns into 3* h_1+1 while "4* h_0+3-- > h_1" indicates that type 4 turns into h_1 because it is odd. Column 12 shows the ratio of the total odd steps to the number of even steps in the iteration. In column 13, the 'YES' indicates that the number of the transformed sequence is less than the initial number N. In column 14 there is the residue in each iteration and where the final residue with which N converges to 1 is **Res_i = 1.2497317.** Note that N=7359 slips to number 2127 at iteration 11 with total steps 69 of which 26 odd passes and 43 even passes and converges to 1 in 30 iterations with total passes 145 of which 51 odd passes and 94 even passes.

Table 2

Collatz sequence using odd numbers N≡1 mod.4 and N≡3 mod.4

Iterations	step	N	Periodicity p-1 or odd and even numbers	Value of w=1+2*k	k odd or even	Total odd	Total even	Value of h_0	Value of h_1	Type of N and its subsequent transformation	Average between odd and even	N'<N	Residue for N-->1
0	0	7359											
1	13	41917	5	115	57	6	7	1839	13972	4*h0+3-->3*h1+1	0,857142857		0,1755830
2	15	10479	0d2p			6	9	10479	10479	4*h0+1-->h0	0,666666667		4,0004958
3	24	26527	3	655	327	10	14	2619	8842	4*h0+3-->3*h1+1	0,714285714		0,3951013
4	35	100723	4	829	414	15	20	6631	33574	4*h0+3-->3*h1+1	0,75		0,2634210
5	39	37771	1	25181	12590	16	23	25180	37771	4*h0+3-->h1	0,695652174		2,6672251
6	44	42493	1	9443	4721	18	26	9442	14164	4*h0+3-->3*h1+1	0,692307692		0,8890721
7	46	10623	0d2p			18	28	10623	10623	4*h0+1-->h0	0,642857143		4,0008833
8	60	30253	6	83	41	24	36	2655	30253	4*h0+3-->h1	0,666666667		0,3512353
9	62	7563	0d2p			24	38	7563	7563	4*h0+1-->h0	0,631578947		4,0010996
10	67	8509	1	1891	945	26	41	1890	2836	4*h0+3-->3*h1+1	0,634146341		0,8891039
11	69	2127	0d2p			26	43	2127	2127	4*h0+1-->h0	0,604651163	SI	4,0012613
12	77	1795	3	133	66	29	48	531	1795	4*h0+3-->h1	0,604166667	SI	1,1854196
13	81	673	1	449	224	30	51	448	673	4*h0+3-->h1	0,588235294	SI	2,6677049
14	84	505	1d2p			31	53	168	168	4*h0+1-->3*h0+1	0,58490566	SI	1,3336048
15	87	379	1d2p			32	55	126	126	4*h0+1-->3*h0+1	0,581818182	SI	1,3342653
16	92	427	1	95	47	34	58	94	142	4*h0+3-->3*h1+1	0,586206897	SI	0,8900973
17	97	481	1	107	53	36	61	106	160	4*h0+3-->3*h1+1	0,590163934	SI	0,8914021
18	100	361	1d2p			37	63	120	120	4*h0+1-->3*h0+1	0,587301587	SI	1,3388428
19	103	271	1d2p			38	65	90	90	4*h0+1-->3*h0+1	0,584615385	SI	1,3397706
20	111	229	3	17	8	41	70	67	229	4*h0+3-->h1	0,585714286	SI	1,1920068
21	113	57	0d2p			41	72	57	57	4*h0+1-->h0	0,569444444	SI	4,0290710
22	116	43	1d2p			42	74	14	14	4*h0+1-->3*h0+1	0,567567568	SI	1,3371589
23	121	49	1	11	5	44	77	10	16	4*h0+3-->3*h1+1	0,571428571	SI	0,8966524
24	124	37	1d2p			45	79	12	12	4*h0+1-->3*h0+1	0,569620253	SI	1,3623556
25	126	9	0d2p			45	81	9	9	4*h0+1-->h0	0,555555556	SI	4,1148699
26	129	7	1d2p			46	83	2	2	4*h0+1-->3*h0+1	0,554216867	SI	1,3345524
27	136	13	2	1	0	49	87	1	4	4*h0+3-->3*h1+1	0,563218391	SI	0,6151023
28	138	3	0d2p			49	89	3	3	4*h0+1-->h0	0,550561798	SI	4,5693317
29	142	1	1	1	0	50	92	0	1	4*h0+3-->h1	0,543478261	SI	2,8118964
30	145		1d2p			51	94	0	0	4*h0+1-->3*h0+1	0,542553191	SI	1,2497317

Notice that if column 11 contains the transformation in the iteration "4*h_0+1-->3*h_0+1" the number of odd passages increases by 1 unit while that of even passages increases by 2 units so that the difference between even and odd passages increases by 1 unit; if there is transformation in iteration "4*h_0+1-->h_0" the number of odd passages increases by 0 units while that of even passages increases by 2 units so that the difference between even and odd passages increases by 2 units; if there is transformation in iteration "4*h_0+3-->3*h_1+1" with p-1 in the 3rd column, the number of odd passages increases by p units while that of even passages increases by p+1 units, so the difference between even and odd passages increases by 1 unit; finally, if there is transformation in iteration "4*h_0+3-->h_1" With p-1 in the 3rd column, the number of odd passes increases by p-1 units while the number of even passes increases by p+1 units, so the difference between even and odd passes increases by 2 units. So it is true that in each iteration **the difference between the total of even steps (which I indicate with b_i) and the total of odd passages (which I indicate with a_i) It always increases by 1 unit or by 2 units in the next iteration** and this always produces in subsequent iterations a difference between b_i and a_i that always increases. It can also be seen that in column 13 the ratio between the odd and even steps decreases in each iteration if in column 11 there are the two transformations "4*h_0+1-->3*h_0+1" e "4*h_0+1-->h_0" while it increases if in column 11 there are the two transformations "4*h_0+3-->3*h_1+1" e "4*h_0+3-->h_1". Therefore in the sequence are the numbers of type 4*h_0+3 which lengthen the stop time to converge to 1.

Nota 4.1 In the following **Table 3** there is the complete sequence of the same initial number N=7359 that converges to 1 in which some columns of **Table 2** have been considered in order to verify that in each

iteration we have that the difference between b_i and a_i is always equal $T_{1i} + 2*T_{2i}$ i.e. $b_i - a_i = T_{1i} + 2*T_{2i}$. Furthermore, it can be seen from the last column that the trend of the values of $b_i - a_i = T_{1i} + 2*T_{2i}$ with respect to the values of a_i increases with the increase of iterations and in iteration number 11 it becomes greater than the value **m=0.5849625** that we will consider later in **Theorem 6.2**.

Table 3

Successione di Collatz usando numeri solo dispari N≡1 mod.4 e N≡3 mod.4						TOTALI--->	T_{1i}=17	T_{2i}=13	$T_{1i}+2*T_{2i}$=43		
Iterazioni	step =Dispari + Pari	N	Dispari =a	Pari =b	Tipo di N e sua trasformazione successiva	N'<N	4*h0+1-->3*h0+1 oppure 4*h0+3-->3*h1+1 =T_{1i}	4*h0+1-->h1 oppure 4*h0+3-->h1 =T_{2i}	$T_{1i}+2*T_{2i}$	pari - dispari = b-a	trend di T1i+2*T2i rispetto ad a_i
0	0	7359									
1	13	41917	6	7	4*h0+3-->3*h1+1		1	0	1	1	0,167
2	15	10479	6	9	4*h0+1-->h0		0	1	3	3	0,500
3	24	26527	10	14	4*h0+3-->3*h1+1		1	0	4	4	0,400
4	35	100723	15	20	4*h0+3-->3*h1+1		1	0	5	5	0,333
5	39	37771	16	23	4*h0+3-->h1		0	1	7	7	0,438
6	44	42493	18	26	4*h0+3-->3*h1+1		1	0	8	8	0,444
7	46	10623	18	28	4*h0+1-->h0		0	1	10	10	0,556
8	60	30253	24	36	4*h0+3-->h1		0	1	12	12	0,500
9	62	7563	24	38	4*h0+1-->h0		0	1	14	14	0,583
10	67	8509	26	41	4*h0+3-->3*h1+1		1	0	15	15	0,577
11	69	2127	26	43	4*h0+1-->h0	SI	0	1	17	17	0,654
12	77	1795	29	48	4*h0+3-->h1	SI	0	1	19	19	0,655
13	81	673	30	51	4*h0+3-->h1	SI	0	1	21	21	0,700
14	84	505	31	53	4*h0+1-->3*h0+1	SI	1	0	22	22	0,710
15	87	379	32	55	4*h0+1-->3*h0+1	SI	1	0	23	23	0,719
16	92	427	34	58	4*h0+3-->3*h1+1	SI	1	0	24	24	0,706
17	97	481	36	61	4*h0+3-->3*h1+1	SI	1	0	25	25	0,694
18	100	361	37	63	4*h0+1-->3*h0+1	SI	1	0	26	26	0,703
19	103	271	38	65	4*h0+1-->3*h0+1	SI	1	0	27	27	0,711
20	111	229	41	70	4*h0+3-->h1	SI	0	1	29	29	0,707
21	113	57	41	72	4*h0+1-->h0	SI	0	1	31	31	0,756
22	116	43	42	74	4*h0+1-->3*h0+1	SI	1	0	32	32	0,762
23	121	49	44	77	4*h0+3-->3*h1+1	SI	1	0	33	33	0,750
24	124	37	45	79	4*h0+1-->3*h0+1	SI	1	0	34	34	0,756
25	126	9	45	81	4*h0+1-->h0	SI	0	1	36	36	0,800
26	129	7	46	83	4*h0+1-->3*h0+1	SI	1	0	37	37	0,804
27	136	13	49	87	4*h0+3-->3*h1+1	SI	1	0	38	38	0,776
28	138	3	49	89	4*h0+1-->h0	SI	0	1	40	40	0,816
29	142	1	50	92	4*h0+3-->h1	SI	0	1	42	42	0,840
30	145		51	94	4*h0+1-->3*h0+1	SI	1	0	43	43	0,843

In the following **Table 4** there is the sequence of the same initial number N=4*1839+3=7359 adopting the **canonical** Collatz sequence using double steps (i.e. f(N)=(3*N+1)/2) where in column 1 there is the total number of steps, in column 2 the transformed values of N, in column 3 the total of odd steps and in column 4 the total of even steps. Note that the two sequences in Tables 2 and 4 lead to converge to 1 with the same total passages (145), with the same odd passages (51) and even passages (94) for which they are equivalent. It is also noted that all the values of N in the sequence in **Table 4** also result in the values of N in the sequence in **Table 2** with the exception of even numbers and transformations "$4*h_0+1 \rightarrow h_0$" e "$4*h_0+3 \rightarrow h_1$" for "orbit change".

Table 4

Canonical Collatz succession			
step	N	Odd	Even
0	7359	0	0
2	11039	1	1
4	16559	2	2
6	24839	3	3
8	37259	4	4
10	55889	5	5
12	83834	6	6
13	41917	6	7
15	62876	7	8
16	31438	7	9
17	15719	7	10
19	23579	8	11
21	35369	9	12
23	53054	10	13
24	26527	10	14
26	39791	11	15
28	59687	12	16
30	89531	13	17
32	134297	14	18
34	201446	15	19
35	100723	15	20
37	151085	16	21
39	226628	17	22

Go on

Canonical Collatz succession			
step	N	Odd	Even
40	113314	17	23
41	56657	17	24
43	84986	18	25
44	42493	18	26
46	63740	19	27
47	31870	19	28
48	15935	19	29
50	23903	20	30
52	35855	21	31
54	53783	22	32
56	80675	23	33
58	121013	24	34
60	181520	25	35
61	90760	25	36
62	45380	25	37
63	22690	25	38
64	11345	25	39
66	17018	26	40
67	8509	26	41
69	12764	27	42
70	6382	27	43
71	3191	27	44
73	4787	28	45
75	7181	29	46

Go on

Canonical Collatz succession			
step	N	Odd	Even
77	10772	30	47
78	5386	30	48
79	2693	30	49
81	4040	31	50
82	2020	31	51
83	1010	31	52
84	505	31	53
86	758	32	54
87	379	32	55
89	569	33	56
91	854	34	57
92	427	34	58
94	641	35	59
96	962	36	60
97	481	36	61
99	722	37	62
100	361	37	63
102	542	38	64
103	271	38	65
105	407	39	66
107	611	40	67
109	917	41	68
111	1376	42	69
112	688	42	70

Go on

Canonical Collatz succession			
step	N	Odd	Even
113	344	42	71
114	172	42	72
115	86	42	73
116	43	42	74
118	65	43	75
120	98	44	76
121	49	44	77
123	74	45	78
124	37	45	79
126	56	46	80
127	28	46	81
128	14	46	82
129	7	46	83
131	11	47	84
133	17	48	85
135	26	49	86
136	13	49	87
138	20	50	88
139	10	50	89
140	5	50	90
142	8	51	91
143	4	51	92
144	2	51	93
145	1	51	94

5 Numbers N=4*h0 + 3 namely N ≡ 3 mod.4

Important consideration 5.1: The number N=4*h_0+3 is odd so its next is even and therefore is divisible by 2 and we indicate with p its multiplicity by 2 , that is, we assume $N+1=2^p*(1+2*k)$ with **p>=2** and **k≥0** positive integers and therefore $N=2^p*(1+2*k)-1$. For example, if **p=10** we have $N=2^{10}*(1+2*k)-1=2048*k+1023$. It is important to know how many there are N=4*h_0+3 che they lead to a certain multiplicity p for the 2 of N+1 since it is the N of the type 4 that increase the next number in the sequence. The numbers of the type **N=4*h_0+3** have periodicity **p** se $h_0=2^{(p-2)}-1+2^{(p-1)}*k$ for each **p≥2** e **k≥0** Integers. Table **5** below shows how many and what are the values of **h_0** and of the respective **N** that lead to a certain multiplicity **p** in a group of numbers from 0 to $2^{20}-1=1.048.575$ and where the numbers of type N ≡ 3 mod.4 are ¼ of all the 1,048,576 considered. (The numbers N ≡ 0 mod.4 are ¼ of all the 1,048,576 considered, the numbers N ≡ 2 mod.4 are ¼ of all the 1,048,576 considered, the numbers N ≡ 1 mod.4 are ¼ of all the 1,048,576 considered).

Table 5

p=	h_0=	N[0]=	How many out of 1,048,576 numbers	Values of k	As a percentage of 1,048,576 numbers
2	h_0=0+2*k	N[0]=3+8*k	131,072	0<=k<=131.071	12,5000000%
3	h_0=1+4*k	N[0]=7+16*k	65,536	0<=k<=65.535	6,2500000%
4	h_0=3+8*k	N[0]=15+32*k	32,768	0<=k<=32.767	3,1250000%
5	h_0=7+16*k	N[0]=31+64*k	16,384	0<=k<=16.383	1,5625000%
6	h_0=15+32*k	N[0]=63+128*k	8,192	0<=k<=8.191	0,7812500%
7	h_0=31+64*k	N[0]=127+256*k	4,096	0<=k<=4.095	0,3906250%
8	h_0=63+128*k	N[0]=255+512*k	2,048	0<=k<=2.047	0,1953125%
9	h_0=127+256*k	N[0]=511+1024*k	1,024	0<=k<=1.023	0,0976563%
10	h_0=255+512*k	N[0]=1023+2048*k	512	0<=k<=511	0,0488281%
11	h_0=511+1024*k	N[0]=2047+4096*k	256	0<=k<=255	0,0244141%
12	h_0=1023+2048*k	N[0]=4095+8192*k	128	0<=k<=127	0,0122070%
13	h_0=2047+4096*k	N[0]=8191+16384*k	64	0<=k<=63	0,0061035%
14	h_0=4095+8192*k	N[0]=16383+32768*k	32	0<=k<=31	0,0030518%
15	h_0=8191+16384*k	N[0]=32767+65536*k	16	0<=k<=15	0,0015259%
16	h_0=16383+32768*k	N[0]=65535+131072*k	8	0<=k<=7	0,0007629%
17	h_0=32767+65536*k	N[0]=131071+262144*k	4	0<=k<=3	0,0003815%
18	h_0=65535+131072*k	N[0]=262143+524288*k	2	0<=k<=1	0,0001907%
19	h_0=131071+262144*k	N[0]=524287+1048576*k	1	k=0	0,0000954%
20	h_0=262143+524288*k	N[0]=1048575+2097152*k	1	k=0	0,0000954%
		Totale=	262,144	Totale=	25,0000000%

Recall that for any value of p the number N ≡ 3 mod.4 produces in the iteration an increase of odd steps of **p-1** (case 5 of the introduction) **or p** (case 6 of the introduction) while it always produces in the same iteration an increase of even steps of **p+1** for an increase of the difference ***b-a=2*** **or** ***b-a=1*** so for each group of N ≡ 3 mod.4 having the same p we will have that ½ of them are inserted in case 5 of the introduction and ½ of them are inserted in case 6 of the introduction.

It should be noted that the probabilities of having a very high p value are minimal and that the numbers with multiplicity **p=2** are half of the 262,144 numbers of type 4 in the group considered (i.e.131,072 of which 65,536 in case 5 of the introduction and 65,536 in case 6 of the introduction) and that the numbers with multiplicity **p** with **2<=p<=4** of case 5 of the introduction are **10.9375%** out of **25%** of the total in the personalized succession.

6 Proof of the Collatz conjecture.

Note 6.1. Let's consider the number N=4* h0 +3 odd and we know that during the custom sequence the numbers are odd of type 3 or type 4 and that in the custom sequence the difference between the total of even steps (which I indicate with ***b)*** and the total of odd steps (which I indicate with ***a)*** always increases by 1 or 2 units in each iteration so that the value of ***b*** grows more than the value of ***a*****.** Therefore, with the succession of iterations, we have that in an iteration **i** the value of the transformed initial number N **approaches** $N'=$

$(3^{a_i} * N)/2^{b_i}$ and it can be greater or less than the initial number N. The condition for which N'<N occurs when $N' = (3^{a_i} * N)/2^{b_i}$**<N** that is, simplifying for N we have $3^{a_i}/2^{b_i}$**<1** and passing to the logarithm we have $a_i * log(3) - b_i * log(2) < 0$ and that is $a_i < b_i * log(2)/log(3)$ namely a_i/b_i **< 0,6309.** In addition, the following inequalities also arise from the previous inequalities: $b_i > a_i * 1{,}5849625$ and $b_i - a_i > a_i * 0{,}5849625$ equivalent to the previous **one.**

Theorem n.6.1 For every odd number of the type 4, N=4*h₀+3, in the sequence there is at least one iteration in which the initial number N is transformed into a number N' less than N and therefore isopath to it.

Demonstration. Let's consider the number N=4*h₀+3 odd and we know from **Note 6.1** that during the custom sequence the numbers are odd of type 3 or type 4 and that in the custom sequence they are transformed according to the odd steps and the even steps and that since the increase of the even steps is greater than that of the odd steps, there is at least one iteration **i** in which a_i/b_i **< 0,6309** and therefore the value of the initial number **N** is transformed into **N'** and it is less than the initial number N and therefore isopath to it. **c.v.d.**

In fact, in **Table 2** above, we have that at iteration 11 with odd a_{11}=26 and even b_{11}=43 we have N'=2127< N=7359 and a_{11}/b_{11}**= 0,604651163< 0,6309297**. In addition, there is also **43=**$b_{11} > a_{11} * 1{,}5849625 = 41{,}209$ and **17=** $b_i - a_i > a_i * 0{,}5849625 = 15{,}209$.

Note 6.2. It can also be verified that after the transformed number N'<N we always obtain odd numbers M'odd less than the initial number N such that **M'<N'<N** and this always occurs if the numbers following N' are ≡ 1 mod.4 since in them theeven passages increase by 2 units and the odd ones increase by 0 or 1 unit according to cases 3 and 4 of the introduction. In fact, if we have $b_i > a_i * 1{,}5849625$ so what **N'<N** with **N'≡ 1 mod.4** so that in the next iteration **i+1** we have $b_{i+1}=b_i+2$ and $a_{i+1}=a_i+0$ or $b_{i+1}= b_i+2$ and $a_{i+1}= a_i+1$ we still have $b_{i+1} > a_{i+1} * 1{,}5849625$ ($b_{i+1}=b_i+2>(a_i+0) * 1{,}5849625 = a_i * 1{,}5849625$ or $b_{i+1}=b_i+2>(a_i+1) * 1{,}5849625 = a_i * 1{,}5849625 + 1{,}5849625$) so the number **M'** obtained will be less than **N'<N**. If, on the other hand, **N'≡ 3 mod.4** in the next iteration **i+1** based on case 5 of the introduction where **p>=2** we have $b_{i+1}=b_i+p+1$ and $a_{i+1}=a_i+p-1$ whereby $b_{i+1}=b_i+p+1>(a_i+p-1) * 1{,}5849625=a_i * 1{,}5849625+p * 1{,}5849625-1{,}5849625$ so what $b_i+p+1>a_i * 1{,}5849625+p * 1{,}5849625-1{,}5849625$ which is verified if $p+1>=p * 1{,}5849625-1{,}5849625$ and that is if **p<= 4,419** where being **p** integer we have **2<=p<=4**. If, on the other hand, **N'≡ 3 mod.4** in the next iteration **i+1** based on case 6 where **p>=2** of the introduction we have $b_{i+1}=b_i+p+1$ e $a_{i+1}=a_i+p$ whereby $b_{i+1}=b_i+p+1>(a_i+p) * 1{,}5849625=a_i * 1{,}5849625+p * 1{,}5849625$ and therefore $b_i+p+1>a_i * 1{,}5849625+p * 1{,}5849625$ which is verified if $p+1>=p * 1.5849625$ from which **p<=1.709** and this is impossible because **p>=2** for which the inequality $b_{i+1} > a_{i+i} * 1{,}5849625$ is not true and therefore the next number **M'** of **N' in the** sequence will be **M'>N'** and therefore also **M'>N.**

In conclusion, if in iteration **i** we have the number **N'<N** and **N'≡ 1 mod.4** or **N'≡ 3 mod.4** of case 5 with **2<=p<=4** we obtain in the next iteration **i+1** a number **M'** less than N' and N and this situation, if it is repeated again in subsequent iterations, always leads to an increase in the difference between the odd and even steps bringing the initial number N of the sequence closer and closer to 1. If, on the other hand, in iteration **i** we have the number **N'<N** and **N'≡ 3 mod.4** of case 5 with **p>=5 or N'≡ 3 mod.4** of case 6 with p>=2 we obtain in the next iteration **i+1** the number **M'** which returns to being **M'>N'** and therefore also **M'>N** because the relation is not satisfied $b_{i+1} > a_{i+1} * 1{,}5849625$**.** However, in this case the difference between the odd and even steps is increased by 1 or 2 units and the values of ***bi+1*** are increased by 1 or 2 units compared to the increase in the values of ***ai+1***, so by reapplying **Theorem no. 6.1** several timesit will continue to be obtained that the difference between the total of even passages and the total of odd passages always increases by 1 or 2 units. Furthermore, it can be seen from Table 5 that the numbers **N'≡ 1 mod.4** and **N'≡ 3 mod.4** of case 5 with **2<=p<=4** that further decrease the value of the number **N**' in the sequence are **25% + 10.9375% = 35.9375%** of all odd numbers (odd numbers are **50%)** between **0** and $2^n - 1$**for every positive integer n** for which the probability of incurring the sequence in an odd number **N'≡ 3 mod.4 of case 5 with p>=5** and **of case 6 with p>=2 is low** (**14.0625% out of 50% of all odd numbers**) so the value of ***b*** grows more than the value of ***a*** and this decreases the value of N' in each iteration so that it converges more and more to 1.

Corollary theorem 6.1: For every number N of type 1, type 2 and type 3 there is an iteration in which the initial number N is transformed into a number N' less than N and therefore isopath to it.

Demonstration. Herself N=4*h₀ is of the type 1 being an even number it can be divided by 2 and therefore if its multiplicity with respect to 2 is p we can write it $N[0] = 2^p * w$ with **p>=2** and **odd integer w** and therefore for the Collatz sequence after p single steps, in which p is divided times by 2, we have the number **N[p]=w** odd and therefore a number less than the initial number N=4*h₀ with b_1=p even passes and no odd passes, i.e. with b_1=p and a_1=0 so the report is satisfied a_1/b_1**< 0,6309297 namely 0/p=0<0,6309297 with steps t=**a_1+b_1**=p**. Herself $N = 4 * h_0 + 2 = 2(2 * h_0 + 1)$ of type 2 being an even number for the Collatz sequence after the first step, in which it divides by 2, leads to the number $N[1] = 2 * h_0 + 1$ odd and therefore to a

number less than the initial number $N[0] = 4 * h_0 + 2$ with 1 even pass and no odd passes, i.e. with b_1=1 and a_1=0 so the report is satisfied a_1/b_1< 0,6309297 namely 0/1=0<0,6309297 with steps t= a_1+b_1=1. If finally $N = 4 * h_0 + 1$ of type 3 being an odd number for the Collatz sequence after the first double pass becomes $N[2] = (3 * (4 * h_0 + 1) + 1)/2 = 6 * h_0 + 2$ and since it is even in the next step it is divided by 2 and becomes $N[3] = 3 * h_0 + 1$. So if h0 is even then N[3] is odd and is less than the initial number N=4*h_0+1 with 2 even passes and 1 odd pass, i.e. with b_1=2 and a_1=1 so the report is satisfied a_1/b_1< 0,6309297 namely 1/2=0,5<0,6309297 with steps t= a_1+b_1=3, while if h_0 is odd N[3]=h_0 (orbit change) is odd number less than the initial number N=4*h_0+1 with 2 even passes and 0 odd passes, i.e. with b_1=2 and a_1=0 so the report is satisfied a_1/b_1< 0,6309297 namely 0/2=0<0,6309297 with steps t= a_1+b_1=2. c.v.d.

So all 4 types of numbers N mod.4 when the condition a_i/b_i< 0,6309297 (with a_i total odd and b_i total even passes) is verified turns into a number N' less than N and therefore isopath to it. Theorem no. 6.1 can be defined as the "slide" theorem.

Theorem no. 6.2 **Every odd number of the type 4 i.e. N[0]=4* h_0+3 satisfies Collatz's conjecture.**
Demonstration. The number N=4*h_0 +3 is odd and from **Theorem no. 6.1** we know that it slips into an iteration **i** with steps **t=bi+ai** with a_i odd total passes and with b_i total equal passes such that a_i/b_i< 0,6309297. Continuing the succession, according to the different situations that follow one another, the value of b_i always grows compared to the value of a_i (we know that their difference increases by 1 or 2 units for each iteration and for **Note 6.2** the probability that the number N in the sequence after the slide will increase is low). The succession of odd numbers transformed in each iteration depend on the values of a_i and b_i with b_i which always increases compared to a_i so that in a given iteration **s** we get a value of b_s and the corresponding value of a_s with $b_s > a_s$ such that we have $\frac{3^{a_s} * N}{2^{b_s}} < 1$. Moving on to the logarithm of inequality, we have $a_s * \log(3) - b_s * \log(2) + \log(N) < 0$ from which $b_s > a_s * \frac{\log(3)}{\log(2)} + \frac{\log(N)}{\log(2)}$ and from this it follows that $b_s - a_s > a_s * 0.5849625 + \frac{\log(N)}{\log(2)}$ from which being both b_s-a_s who b_s integers we have $b_s = ARR.ECC\left[a_s * \frac{log(3)}{log(2)} + \frac{log(N)}{log(2)}\right]$ and $b_s - a_s = ARR.ECC.\left[a_s * 0,5849625 + \frac{\log(N)}{\log(2)}\right]$ (they are rounded b_s-a_s and b_s to the next integer). So the value b_s-a_s tends to increase compared to at the value of a_s following the course of an increasing line of angular coefficient **m=0.5849625** and of the initial point $Q\left(0; \frac{log(N)}{log(2)}\right)$ con $\frac{log(N)}{log(2)} > 2$ and therefore I can consider in a Cartesian plane (with x-axis the values of a_i and with y-axis the values of b_i-a_i) the function **f:N→N** where **f(a_i)**= $ARR.ECC.\left[a_i * 0,5849625 + \frac{\log(N)}{\log(2)}\right]$= b_i-a_i which has a similar trend to the ascending line $y = a_i * 0,5849625 + \frac{log(N)}{log(2)}$ angular coefficient **m=0,5849625** where the value of y rounds to the next whole number. Furthermore, we recall from **Note 3.1** and **Table 1** of **paragraph 3** and from **Tables 2 and 3 and Note 4.1** of **paragraph 4** that in each iteration **i** the difference b_i-a_i is equal to T_{1i} +2*T_{2i} whose value depends on the value of a_i and always increases with the increase of iterations with a tendency that becomes greater than **m=0.5849625** so it tends to meet with the line of the function **f(a_i)** for a certain value of a_i. Therefore we will have in an iteration **s** such that
T_{1s} +2*T_{2s}= b_s-a_s= $ARR.ECC.\left[a_s * 0,5849625 + \frac{\log(N)}{\log(2)}\right]$= f($a_s$) so that in this iteration **s** the initial value of **N** is positive and is less than **1** for which $(3^{a_s} * N * Res)/(2^{b_s}) = 1$. Furthermore, in all subsequent iterations to **s** we will have that for every positive integer **i** b_{s+i} - a_{s+i} =1 where b_{s+i} increases by 2 units and a_{s+i} increases by 1 unit in all iterations **s+i** subsequent to **s.** Therefore any odd number of the type 4 i.e. N[0]=4* h_0+3 satisfies Collatz's conjecture. **c.v.d.**

Theorem 6.3 **Every even number of type 1 and type 2 and every odd number of type 3 satisfies Collatz's conjecture.**
Demonstration. The number type 3 N[0]=4* h_0 +1 is odd and by the **corollary to Theorem 6.1** it slips into an odd number N' which continues its sequence in the same way as the number type 4 (after the first step) and therefore for **Theorem 6.2** in the same way satisfies the conjecture. The number type 1 N[0]=4* h_0 +0 it is even and by the **corollary to Theorem No. 6.1** it slips into an odd number N' and therefore will continue its sequence in the same way as we have seen before and therefore for **Theorem No. 6.2** in the same way it satisfies the conjecture. The type 2 number N[0]=4* h_0 +2 is even and by the **corollary to Theorem No. 6.1** it slips into an odd number N' and continues its sequence in the same way as we have seen before and therefore for **Theorem No. 6.2** in the same way it satisfies the conjecture.

Corollary of theorem no. 6.2 and no. 6.3 Each number N mod.4 continues the sequence after convergence to 1 (for Theorems no. 6.2 and no. 6.3) with the cycle 1 – 4 – 2 – 1.

Demonstration. From **Theorems 6.2 and 6.3** we have that N converges to 1 because there is the number of odd steps a_s and there is an even number of passages $b_s = ARR.ECC\left[a_s * \frac{log(3)}{log(2)} + \frac{log(N)}{log(2)}\right]$ whereby b_s-a_s= $ARR.ECC.\left[a_s * 0,5849625 + \frac{\log(N)}{\log(2)}\right] = T_{1s} + 2 * T_{2s}$. The next cycle 1 – 4 – 2 – 1 is obtained with an odd step followed by 2 even passes for which the value of a_s=a_s **+1** e b_{s+1}=b_s **+2.** Calculate $b_{s+1} = ARR.ECC.\left[(a_{s+1}) * \frac{\log(3)}{\log(2)} + \frac{\log(N)}{\log(2)}\right]$=$ARR.ECC.\left[a_s * \frac{\log(3)}{\log(2)} + \frac{\log(N)}{\log(2)}\right]$+$ARR.ECC.\left[\frac{\log(3)}{\log(2)}\right] = b_s + 2$ dove $b_s = ARR.ECC\left[a_s * \frac{log(3)}{log(2)} + \frac{log(N)}{log(2)}\right]$ e $ARR.ECC.[log(3)/log(2)] = 2$ per cui b_{s+1}= b_s+2 e questo vale per ogni ciclo 1 – 4 – 2 – 1 successivo. **C.v.d.**

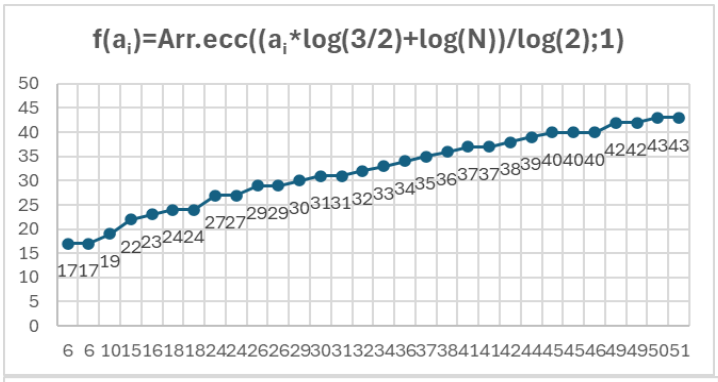
We can verify **Theorem 6.2** above using **Table 6** below (also Table **2** above) where for each iteration **i** have been inserted in the respective columns the values of a_i, the corresponding values of $b_i = ARR.ECC\left[a_i * \frac{log(3)}{log(2)} + \frac{log(N)}{log(2)}\right]$ and function values **y= a_i*0,5849625+log(N)/log(2), y= f(a_i)= ARR.ECC.** $[a_i * 0,5849625 + log(N)/log(2)]$ and the values of T_{1i} **+2***T_{2i}.

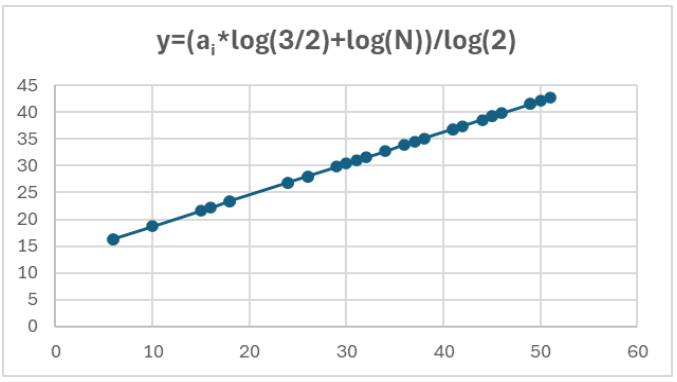
Table 6

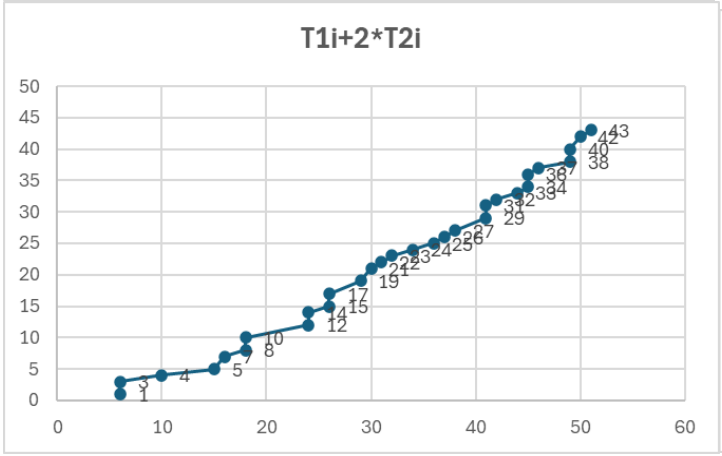
Successione di Collatz usando numeri solo dispari N≡1 mod.4 e N≡3 mod.4						TOTALI--->	T_{1i}=17	T_{2i}=13	T_{1i}+2*T_{2i}=43				
Iterazioni	step =Dispari + Pari	N	Dispari=a	Pari=b	Tipo di N e sua trasformazione successiva	N'<N	4*h0+1-->3*h0+1 oppure 4*h0+3-->3*h1+1 =T_{1i}	4*h0+1-->h1 oppure 4*h0+3-->h1 =T_{2i}	T_{1i}+2*T_{2i}	Step pari -step dispari = b-a	y=(a_i*log(3/2)+log(N))/log(2)	f(a_i)=Arr.ecc((a_i*log(3/2)+log(N))/log(2);1)	Verifica T_{1i}+T_{2i}=Arr.ecc.((a_i*log(3/2)+log(N))/log(2);1)
0	0	7359											
1	13	41917	6	7	4*h0+3-->3*h1+1		1	0	1	1	16,35506902	17	NO
2	15	10479	6	9	4*h0+1-->h0		1	1	3	3	16,35506902	17	NO
3	24	26527	10	14	4*h0+3-->3*h1+1		2	1	4	4	18,69491903	19	NO
4	35	100723	15	20	4*h0+3-->3*h1+1		3	1	5	5	21,61973153	22	NO
5	39	37771	16	23	4*h0+3-->h1		3	2	7	7	22,20469403	23	NO
6	44	42493	18	26	4*h0+3-->3*h1+1		4	2	8	8	23,37461903	24	NO
7	46	10623	18	28	4*h0+1-->h0		4	3	10	10	23,37461903	24	NO
8	60	30253	24	36	4*h0+3-->h1		4	4	12	12	26,88439404	27	NO
9	62	7563	24	38	4*h0+1-->h0		4	5	14	14	26,88439404	27	NO
10	67	8509	26	41	4*h0+3-->3*h1+1		5	5	15	15	28,05431904	29	NO
11	69	2127	26	43	4*h0+1-->h0	SI	5	6	17	17	28,05431904	29	NO
12	77	1795	29	48	4*h0+3-->h1	SI	5	7	19	19	29,80920654	30	NO
13	81	673	30	51	4*h0+3-->h1	SI	5	8	21	21	30,39416904	31	NO
14	84	505	31	53	4*h0+1-->3*h0+1	SI	6	8	22	22	30,97913154	31	NO
15	87	379	32	55	4*h0+1-->3*h0+1	SI	7	8	23	23	31,56409404	32	NO
16	92	427	34	58	4*h0+3-->3*h1+1	SI	8	8	24	24	32,73401904	33	NO
17	97	481	36	61	4*h0+3-->3*h1+1	SI	9	8	25	25	33,90394405	34	NO
18	100	361	37	63	4*h0+1-->3*h0+1	SI	10	8	26	26	34,48890655	35	NO
19	103	271	38	65	4*h0+1-->3*h0+1	SI	11	8	27	27	35,07386905	36	NO
20	111	229	41	70	4*h0+3-->h1	SI	11	9	29	29	36,82875655	37	NO
21	113	57	41	72	4*h0+1-->h0	SI	11	10	31	31	36,82875655	37	NO
22	116	43	42	74	4*h0+1-->3*h0+1	SI	12	10	32	32	37,41371905	38	NO
23	121	49	44	77	4*h0+3-->3*h1+1	SI	13	10	33	33	38,58364405	39	NO
24	124	37	45	79	4*h0+1-->3*h0+1	SI	14	10	34	34	39,16860655	40	NO
25	126	9	45	81	4*h0+1-->h0	SI	14	11	36	36	39,16860655	40	NO
26	129	7	46	83	4*h0+1-->3*h0+1	SI	15	11	37	37	39,75356905	40	NO
27	136	13	49	87	4*h0+3-->3*h1+1	SI	16	11	38	38	41,50845655	42	NO
28	138	3	49	89	4*h0+1-->h0	SI	16	12	40	40	41,50845655	42	NO
29	142	1	50	92	4*h0+3-->h1	SI	16	13	42	42	42,09341906	43	NO
30	145		51	94	4*h0+1-->3*h0+1	SI	17	13	43	43	42,67838156	43	SI

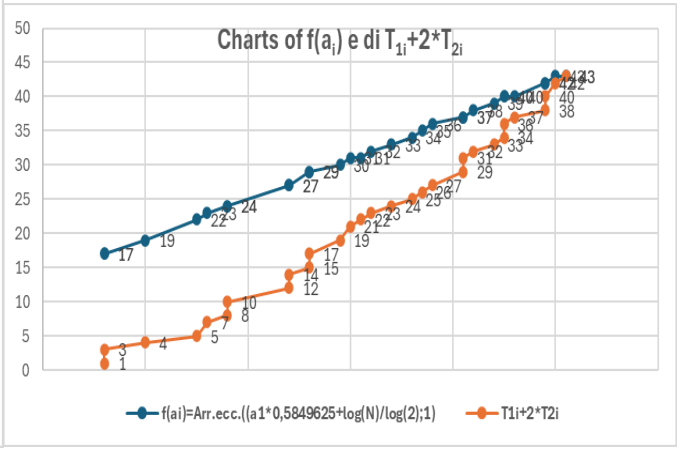
In fact, from **Table 2** and **Table 6** we have at the iteration **s=30** with total steps a_s+b_s **=145** (odd steps a_s **= 51** and even b_s **= *94***) we have **N[145]=1** and we verify that b_s**=ARR.ECC.[*51** 1.5849625+log(N)/log(2)]=94** and we have that a_s+b_s **=ARR.ECC.[*51** 2,5849625+log(N)/log(2)]=145** and we also have b_s-a_s = **f(a_s)= ARR.ECC.[*51**0,5849625+log(N)/log(2)]=** T_{1i} **+2***T_{2i} **= 43**. This result is only obtained in the latest iteration of **Table 6**. In fact, if we consider the iteration **i=23** with a_{23} **=44** and b_{23} **=77** we have that b_{23}-a_{23} **= ARR.ECC.[*44**0,5849625+log(N)/log(2)]=39** other than **33= 77-44**. Then the initial number N=7359 converges to 1 with total steps a_s+b_s=**145** so the conjecture is verified.

We can also verify the previous **Theorem 6.2** using Cartesian diagrams by placing the values of the odd passages ai on the x-axis (column 4 of Table 6) and the values on the y-axis T_{1i} **+2***T_{2i} , values b_i-a_i of the differences between even and odd steps of functions **y= a_i*0,5849625+log(N)/log(2), y= f(a_i)= ARR.ECC.[a_i*0,5849625+log(N)/log(2)]** (columns 10 - 11 - 12 - 13 of Table 6).









From the last diagram you can see how the graphs of **f(*a*$_i$)** and values **T$_{1i}$ +2*T$_{2i}$** meet at the point where odd passages ***a*$_s$ =51** and even ***b*$_s$ =94** the initial number **N satisfies the conjecture.**
Many tests have been carried out on different numbers containing even a number of numerous digits using a python program of my creation with numbers having many digits that have confirmed the veracity of the conjecture.

Conclusions

If you start the sequence with an even number ≡ 0 mod.4 or ≡ 2 mod.4 for cases 1. and 2. of the introduction, you will have in the next iteration an odd number ≡ 1 mod.4 or ≡ 3 mod.4 which I will call N1 and will be of the type N1=4*w+1 or N1=4*w+3 and then you will proceed in the subsequent iterations with always odd numbers ≡ 1 mod.4 or ≡ 3 mod.4 following the cases 3. - 4. - 5. - 6. of the introduction. The difference between the total *b* of the even steps and the total *a* of the odd steps always increases by 1 or 2 units for each iteration and this produces in subsequent iterations a difference between *b* and *a* that always increases so that, in a given subsequent iteration **i** with total even steps of ***b*$_i$** and with total odd passages of ***a*$_i$**, linked together by the relationship ***a*$_i$/*b*$_i$<Log(2)/Log(3)** namely ***a*$_i$/*b*$_i$<0,63092975***,* the number obtained is less than the initial number **N** (we have the so-called "slide").

Continuing further, we arrive at a single subsequent iteration in which the total values pass by ***b*$_s$** and the total odd steps values of ***a*$_s$** with ***b*$_s$ > *a*$_s$**, linked together by the following three reports:

a) ***b*$_s$=ARR.ECC.[*a*$_s$*1,5849625+log(N)/log(2)]** where ***b*$_s$** *rounds to the next whole number*
b) ***b*$_s$ - *a*$_s$=ARR.ECC.[*a*$_s$*0,5849625+log(N)/log(2);1]** where ***b*$_s$ - *a*$_s$** *rounds to the next whole number*
c) ***b*$_s$+*a*$_s$=ARR.ECC.[*a*$_s$*2,5849625+log(N)/log(2);1]** where ***b*$_s$+*a*$_s$** *rounds to the next whole number*

are such that the number obtained at iteration **s** is equal to 1 for which the conjecture is verified. Furthermore, the sequence always continues with the cycle 1 – 4 – 2 – 1 in which the previous relationship a) is respected.

References

[1] Eric Roosendaal, You http://www.ericr.nl/wondrous/index.html a problem 3x + 1,

[2] Wei Ren, Simin Li, Ruiyang Xiao and Wei Bi, The Collatz conjecture for 2100000 −1 is true- Algorithms for verifying extremely large numbers, IEEE UIC Proc., October 2018, Guangzhou, China, 411-416, 2018

Contacts

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